

Tables and Formulas

to accompany

The Use of School Means as Variables

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Notation:

N = total number of cases (e.g. number of students)

m = number of subgroups (e.g. number of schools)

n_i = number of cases in subgroup i

n = average number of cases per subgroup

r_{AB} = correlation between variables A and B, based on N cases

$\bar{r}_{\bar{A}\bar{B}}$ = correlation between subgroup means, based on m pairs of subgroup means, with each pair weighted by the appropriate n_i .

r_{ab} = within-groups correlation of A and B, i.e. correlation between the deviations of scores on A and B from their subgroup means. It is based on N cases.

r_{AA} , r_{aa} , and $\bar{r}_{\bar{A}\bar{A}}$ are the corresponding reliability coefficients

\hat{r} = r corrected for shrinkage

s_A and s_B are standard deviations of variables A and B, for the total group (N cases)

$\bar{s}_{\bar{A}}$ and $\bar{s}_{\bar{B}}$ are the standard deviations of subgroup means, with each of the m means weighted by the appropriate n_i .

s_a and s_b are within-group standard deviations, i.e. standard deviations of the deviations of scores on A and B from their subgroup means. These standard deviations are based on N cases.

σ_A , σ_B , σ_a , σ_b , $\bar{\sigma}_{\bar{A}}$ and $\bar{\sigma}_{\bar{B}}$ are the unbiased estimates of the population values corresponding to s .

$$k_A = \frac{s_a}{\bar{s}_{\bar{A}}}$$

$$k_B = \frac{s_b}{\bar{s}_{\bar{B}}}$$

$$n = \frac{N}{m}$$

Formulas* for estimating r_{AB}^-

$$r_{AB}^- = \frac{r_{AB} s_A s_B - r_{ab} s_a s_b}{s_A s_B} \quad (2)$$

$$r_{AB}^- = r_{AB} + k_A k_B (r_{AB} - r_{ab}) \quad (3)$$

Angoff Formula #16**, for correcting split-half reliability coefficients

$$r_{AA} = \frac{r_{12} s_A^2}{(s_1 + r_{12} s_2)(s_2 + r_{12} s_1)} \quad (4)$$

where r_{12} is the correlation between half-tests, and s_1 and s_2 are the half-test standard deviations

Formulas * for estimating r_{AA}^-

$$r_{AA}^- = r_{AA} + k_A^2 (r_{AA} - r_{aw}) \quad (5)$$

$$\hat{r}_{AA}^- = 1 - \frac{\sigma_A^2}{n \sigma_{\bar{A}}^2} (1 - r_{AA}) \quad (6')$$

$$r_{AA}^- = 1 - \left(1 + k_A^2\right) \frac{1 - r_{AA}}{n} \quad (6)$$

*Formula (2) is an exact formula. Formulas (3), (5), (6') and (6) represent varying degrees of approximation.

**Angoff, W.E. Test reliability and effective test length, Psychometrika, 1953, 18, 1-14.

Table 1. Reliability coefficients for students, school means, and students-within-school

For TALENT Information Part II total score and 14 of the subscores
Based on 17949* Grade 12 boys, in 814 secondary schools

Info. Part II score	No. of items	Standard deviations			k _A	Reliability estimates					
		s _A	s _a	s _Ā		r _{AA} (4)	r _{aa} (4)	r _{ĀĀ} (4)	r _{ĀĀ} (5)	r _{ĀĀ} (6)	
R-192 Total	143	17.90	15.49	8.98	1.72	.923	.904	.978	.980	.99	
R-131 Art	12	2.47	2.16	1.19	1.82	.655	.583	.893	.895	.93	
R-132 Law	9	1.74	1.58	.74	2.12	.523	.459	.835	.810	.88	
R-133 Health	9	1.76	1.59	.76	2.10	.554	.487	.847	.850	.89	
R-134 Engineering	6	1.12	1.05	.40	2.62	.377	.325	.725	.734	.78	
R-135 Architecture	6	1.40	1.29	.55	2.34	.423	.360	.760	.769	.83	
R-137 Foreign travel	5	1.31	1.16	.60	1.92	.527	.457	.791	.785	.90	
R-138 Military	7	1.49	1.37	.57	2.42	.480	.426	.793	.797	.84	
R-139 Acct., Bus., sales	10	1.94	1.74	.86	2.04	.563	.498	.844	.833	.90	
R-142 Bible	15	3.38	3.08	1.39	2.21	.773	.746	.901	.905	.94	
R-145 Hunting	5	1.28	1.14	.57	2.00	.455	.362	.822	.825	.88	
R-146 Fishing	5	1.31	1.21	.49	2.45	.476	.455	.633	.602	.83	
R-147 Outdoor activ. (other)	9	1.72	1.56	.72	2.16	.478	.408	.805	.806	.86	
R-150 Theater, ballet	8	1.76	1.56	.81	1.93	.585	.523	.847	.815	.91	
R-162 Vocab. II	9	1.95	1.73	.89	1.94	.657	.602	.883	.864	.93	

*Only cases in the one-year follow-up are included.

Table 2. Intercorrelations* among 10 TALENT variables, for students, school means, and students-within-school
Based on 17949** Grade 12 boys, in 814 secondary schools.

	No. of items	att										s	k							
		R-230	R-290	R-312	R-333	R-190	R-192	R-105	R-107	R-145	R-146									
1. R-230	113																			
Eng		r ^{AB}	.468	.645	.535	.636	.605	.554	.547	.036	.128	.393	12.67							
Tot		r ^{ab}	.407	.601	.489	.578	.540	.485	.498	.048	.113	.346	11.41							
		r ^{AB}	.752	.825	.730	.849	.845	.810	.756	-.014	.204	.599	5.51 2.07							
2. R-290	15																			
Abst		r ^{AB}		.530	.458	.538	.478	.428	.490	.063	.121	.299	2.81							
Reas		r ^{ab}		.478	.408	.463	.396	.345	.434	.086	.111	.255	2.57							
		r ^{AB}	.750	.767	.686	.808	.764	.755	.755	-.038	.175	.510	1.15 2.23							
3. R-312	24																			
Math		r ^{AB}		.782	.782	.721	.592	.576	.686	.016	.118	.484	5.69							
II		r ^{ab}	.826	.752	.752	.677	.528	.512	.651	.053	.128	.446	5.08							
		r ^{AB}		.906	.906	.876	.819	.803	.833	-.132	.072	.645	2.55 1.99							
4. R-333	14																			
Math		r ^{AB}		.728	.728	.638	.523	.493	.611	-.009	.100	.438	3.31							
III		r ^{ab}	.684	.684	.591	.458	.423	.423	.572	.046	.126	.390	2.97							
Tot		r ^{AB}	.839	.869	.799	.755	.743	.743	.773	-.232	-.025	.649	1.46 2.03							
5. R-190	252																			
Inf I		r ^{AB}		.821	.821	.868	.822	.822	.869	.162	.278	.458	36.22							
Tot		r ^{ab}	.794	.812	.750	.792	.792	.792	.859	.214	.292	.426	31.17							
		r ^{AB}	.837	.869	.799	.806	.755	.743	.773	-.012	.235	.582	18.44 1.69							
6. R-192	143																			
InfII		r ^{AB}		.821	.821	.868	.822	.822	.869	.162	.278	.458	36.22							
Tot		r ^{ab}	.794	.812	.750	.792	.792	.792	.859	.214	.292	.426	31.17							
		r ^{AB}	.837	.869	.799	.806	.755	.743	.773	-.012	.235	.582	18.44 1.69							
7. R-105	24																			
Soc.		r ^{AB}		.754	.754	.911	.878	.878	.830	-.101	.107	.533	2.30 1.76							
stud		r ^{ab}	.806	.801	.743	.911	.878	.878	.830	-.101	.107	.533	2.30 1.76							
		r ^{AB}	.806	.801	.743	.911	.878	.878	.830	-.101	.107	.533	2.30 1.76							
8. R-107	18																			
Phys.		r ^{AB}		.749	.749	.904	.825	.823	.830	-.059	.150	.562	1.87 2.08							
Sci.		r ^{ab}	.758	.831	.775	.904	.825	.823	.830	-.059	.150	.562	1.87 2.08							
		r ^{AB}	.758	.831	.775	.904	.825	.823	.830	-.059	.150	.562	1.87 2.08							
9. R-145	5																			
Hunt		r ^{AB}		-.039	-.231	-.013	-.032	-.100	-.062		.383	-.036	1.28							
Inf.		r ^{ab}	-.014	-.131	-.231	-.013	-.032	-.100	-.062		.383	-.036	1.28							
		r ^{AB}	-.014	-.131	-.231	-.013	-.032	-.100	-.062		.383	-.036	1.28							
10. R-146	5																			
Fish		r ^{AB}	.204	.176	.069	.220	.265	.102	.152	.642	.069	.069	1.31							
Inf.		r ^{ab}	.204	.176	.069	.220	.265	.102	.152	.642	.069	.069	1.31							
		r ^{AB}	.204	.176	.069	.220	.265	.102	.152	.642	.069	.069	1.31							

Table 2 cont'd.

- * Upper right half of table contains exact values. Lower left half contains r_{AB} estimates based on Formula (3).
- ** Only cases in the one-year follow-up are included.
- *** Point biserial r with entry into college.